
Golden Ratio

ID: 13636

Time required
45 minutes

Activity Overview

In this activity, students will investigate Fibonacci numbers and the ratios of successive Fibonacci numbers. They will take measurements of their height and their navel heights and investigate the relationship between the two measurements. Finally, they will examine a scatter plot and a linear relationship fitted to the data points.

Topic: Number and Operations

- *Sequences, Patterns*
 - *Fibonacci numbers, Golden Ratio*
 - *Ratio, Proportion*
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Teacher Preparation and Notes

- *Students will need to be familiar with the vocabulary terms ratio, proportion, Fibonacci numbers, and Golden ratio.*
- *TI-Navigator is not required for this activity, but an extension is given for those teachers that would like to use it.*
- ***To download the student worksheet and TI-Navigator file, go to education.ti.com/exchange and enter "13636" in the quick search box.***

Associated Materials

- *MGAct21_GoldenRatio_worksheet_TI73.doc*
- *MGAct21_GoldenRatio_Nav_TI73.act*

Suggested Related Activities

To download the activity listed, go to education.ti.com/exchange and enter the number in the quick search box.

- *Class of Gold (TI-73 Explorer) — 4476*
- *The Golden Ratio (TI-84 Plus family & TI-Navigator) — 7578*
- *Golden Rectangles (TI-15 Explorer) — 4984*

Problem 1 – Fibonacci Numbers

In the first part of this lesson, students will explore the Fibonacci sequence. Students should have some exposure to the Fibonacci numbers prior to this activity.

Questions 1–3

After students have the first 20 terms of the Fibonacci numbers, have them enter them into L1 on the TI-73. Press **LIST** and enter them in L1. Then in L2, they can either copy L1 as a shortcut or enter the 2nd through 21st terms in L2. To copy L1, move to the top of L2 and press **2nd** **LIST** **1** **ENTER**. Then move to the first element and press **DEL**. To find the ratio, enter **L1/L2** at the top of L3. Move to the top of L3 and press **2nd** **LIST** **2** **÷** **2nd** **LIST** **1** **ENTER**.

L1	L2	L3
1	1	1
1	2	2
2	3	1.5
3	5	1.6667
5	8	1.6
8	13	1.625
13	21	1.6154
L3 = L2 / L1		

Problem 2 – Height Ratio – Gathering the Data

Next, students will explore the Golden ratio in measurements of heights. Have students work in pairs to measure the height from floor to head and from floor to belly button.

Question 4

After students complete the table for their measurements, collect all of the class data into a table. Then, students can enter the data in lists or you can link the lists around from student to student.

To have students enter the data, move to the top of an empty list or insert a new list after L3 (**2nd** **DEL** at the top of the column). To name the list, press **2nd** **MATH**, select the letters, highlight DONE and press **ENTER**, and press **ENTER** to store the list. Now you can use it like you use L1-L6.

L5	L6	L7
-----	-----	
Name=HT		

Dividing **HT** by **NHT** can be done like in Question 3. Just select the appropriate list name after pressing **2nd** **LIST**.

Questions 5–6

Students can find this information by scrolling down through the **HTNHT** list. They will need to compare the entries to the Golden ratio.

Question 7

Begin this calculation from a clear Home screen. To find the mean(function, press **2nd** **LIST** and arrow **▶** over to MATH. Then, press **2nd** **LIST** again and arrow down **▼** to the list name.

```
mean(LHTNHT)
1.621174076
```

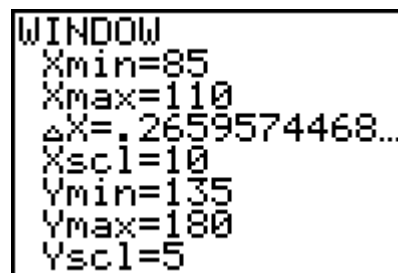
Questions 8–9

Students can graph the data by looking at the elements in the lists or by looking at the class data table. There should be an obvious trend that the taller the height, the taller the navel height.

Problem 3 – The Best Fit Line

Questions 10–11

For students to graph the scatter plot, they will need to determine an acceptable window. Press **WINDOW** and enter appropriate numbers for the class data. Use the class table of data to set an appropriate window for your data.



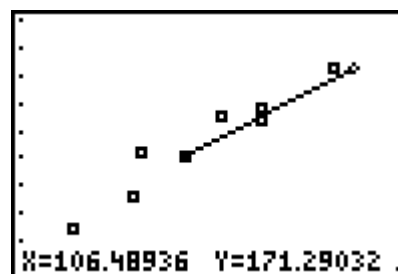
The discussion around selecting a window will help students better understand their data as well as help in future situations when they need to select a window on their own.

Questions 12–14

After students have graphed the scatter plot, they can use the **Manual-Fit** command to draw a line of best fit. From a clear Home screen, press **2nd** **LIST** **◀** to access the **CALC** menu.

Manual-Fit will be option 3. Select **3** to copy it to the Home screen.

Then, press **2nd** **APPS** **2** (to select Y= variables) and **1** (to select Y1). Once you press **ENTER**, the scatter plot will reappear. Move the cursor to where you want the first point on your equation to lie. Press **ENTER** to mark the first point. Then, move the cursor to where you want the second point on the line to be and press **ENTER**.



After the line is drawn, you can use the arrow keys to move the line around until you are happy with the location for the best-fit line. Press **ENTER** one more time and it will store the equation to the Y= menu.

To see the equation, press **Y=**. All students' equations will vary but most should be relatively close.

Questions 15–19

Have students enter the two additional equations in Questions 16 and 17. They can compare these equations to the equation they found in Question 13. Students should find that the equations are similar but not exactly the same. Discuss the differences as a class.

Discuss with students which equations would best answer Questions 18 and 19 and have students explain their reasoning.

Extension – TI-Navigator™

1. To gather the student data, use **MGAct21_GoldenRatio_Nav_T173.act**. Open in Activity Center and start the activity. Students will need to input their **HT** and **NHT** values into the lists that appear on their individual calculator.

After all students have input their data, stop the activity. Then configure the activity to use “Existing Activity Lists” and then start the activity again. This will send the existing lists back to the students’ calculators for use throughout the activity.

If desired, you could also calculate **NHTHT** and send it to the class. This will ensure all students are working with the same data to answer the rest of the activity’s questions.

2. Use **Screen Capture** throughout to monitor student progress.
3. Use **QuickPoll** to have students determine if two measurements (or numbers) are or are not in proportion to the Golden ratio. For example, open a **Yes/No** poll and call out two numbers. Students should determine if the numbers do or do not have the Golden ratio and select Yes or No to answer the poll.

Solutions – student worksheet
Problem 1

1. 21, 34, 55, 89, 144, 233, 377, 610, 987, 1597, 2584, 4181, 6765
2. 10,946
3. 1, 0.5, 0.66667, 0.6, 0.625, 0.61538, 0.61905
4. Answers will vary. Check students' work for accuracy.
5. Answers will depend on class data. The student ratio should be closest to 1.618.
6. Answers will depend on class data. The student ratio should be farthest from 1.618.
7. Answer will depend on class data.
8. Check students' work. Graph should generally show a steady increase of Height as Navel Height increases.
9. Students should conclude that there is a steady increase of Height as Navel Height increases.

Problem 2

10. Windows will vary based on class data. In general, the X_{min} and Y_{min} should be just below the least value in the list and X_{max} and Y_{max} should be just above the greatest values in the list.
11. Check students' work. Graph should appear much like the graph from Question 8.
12. Check students' work. Line of fit should be reasonable based on the data in the graph.
13. Answers will vary. Equation should match the graph found.
14. The value in front of the x -variable should be relatively close to the Golden ratio, 1.618.
15. $Y = 1.618X$
16. The graphs of Y_1 and Y_2 should be similar.
17. Again, the graphs of Y_1 , Y_2 , and Y_3 should be close together. The slope and/or y -intercepts may be slightly different but related.
18. Y_2 , or $Y = 1.618X$.
19. 161.8 cm



Problem 1 – Fibonacci Numbers

1. The first seven terms in the Fibonacci sequence are 1, 1, 2, 3, 5, 8, and 13. Write the next 13 terms. _____

2. Enter the first 20 terms in L1 and the 2nd through 21st terms in L2. Press [LIST] and move to the top of L1. Press [CLEAR] [ENTER] if needed to empty the list. After entering L1, move to the very top of L2. Press [CLEAR] [ENTER] and then insert L1 (2nd [LIST] [ENTER] [ENTER]). Then delete the first element by highlighting it and pressing [DEL]. Then enter the 21st element at the end of the list.

L1	L2	L3	2
1	1	-----	
1	2		
2	3		
3	5		
5	8		
8	13		
13	21		
L2(1)=1			

In L3, enter the ratio between L1 and L2. Go to the top of L3 and press [2nd] [LIST] [ENTER] [2nd] [LIST] [2] [ENTER] to enter **L1/L2** and execute the command. What are the first 6 ratios? _____

3. Describe the pattern you see in the ratios as you move down the list. _____

Problem 2 – Height Ratio – Gathering the Data

Next, you will explore the ratio between your height and the height of your navel.

4. Record your measurements in the table below.

	Height (cm) LHT	Navel Height (cm) LNHT	Height/Navel Height LHTNHT
Me			
Partner			

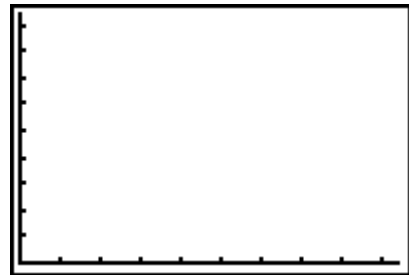
Now collect all the data from the class and enter in the indicated lists. Press [LIST] and insert lists ([2nd] [DEL]) with the given names. Find **HTNHT** by dividing the first list, **HT**, by the second list, **NHT**.

5. Which student's ratio is the most "golden"? Justify your answer with a reason.

6. Which student's ratio is the least "golden"? Justify your answer with a reason.

7. Find the mean average of the Height / Navel Height list (LHTNHT). Use **Mean(**, found in the **Stat > MATH** menu. Press [2nd] [LIST] [▶] [▶] [3] and then [2nd] [LIST] to find **HTNHT**.

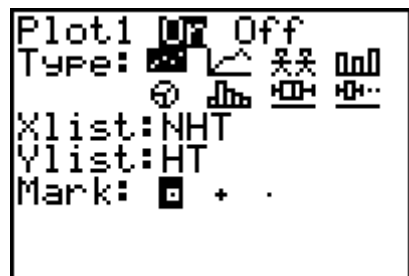
8. Sketch the navel height versus the student height. Make sure to label the axes and use numbers to show the values on the axes.
9. In general, as navel height increases, what happens to the student height? _____



Problem 3 – The Best Fit Line

Now, we are going to graph the scatter plot of data and find a best fit line that models the data.

10. Set up and graph the scatter plot of navel height versus height. The screen at the right shows the settings for the scatter plot. Press 2^{nd} $Y=$ $ENTER$ to access **Plot1**. To access the list names, press 2^{nd} $LIST$ and move down to the appropriate name.

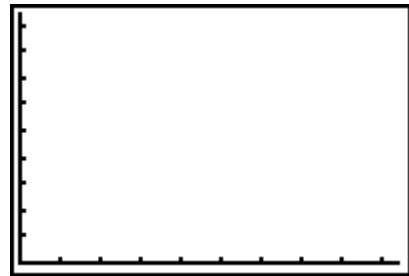


What would be an appropriate window for the data for your class?

Xmin? _____ Xmax? _____ Xscl? _____
 Ymin? _____ Ymax? _____ Yscl? _____

Press 2^{nd} $WINDOW$ to enter the values.

11. Press $GRAPH$ and sketch the scatter plot of the class data to the right.



12. Next, find a line of fit for the data using the **Manual-Fit** command. From the Home screen, press 2^{nd} $LIST$ \leftarrow 3 (to select Manual-Fit) and 2^{nd} $APPS$ 2 1 (to select Y1), then press $ENTER$. When the graph appears again, move the cursor to the first end point that you want for the line of fit. Press $ENTER$. Move to the other end point and press $ENTER$ again. After the line is drawn, you can use the arrows to move the line about. (up and down or to adjust the slope) Press $ENTER$ one last time when you are happy with where the line is located.

Sketch the line of fit on the scatter plot in Question 11.

13. What is the equation found? Press $Y=$ to see the Y1 equation. _____
14. The number in front of the X in the equation is defined as the ratio of change in y over change in x . How does this number compare to the Golden ratio from Question 3?



15. If $Y = \text{Height}$, $X = \text{Navel Height}$, and $Y \div X = 1.618$, then $Y =$ _____.

16. Type the equation from Question 15 in Y_2 and press **[GRAPH]**. Are there similarities between Y_1 and Y_2 ? If so, explain. _____

17. Next, use the mean average of the class to write an equation in Y_3 . In Y_3 , type in the number you found in Question 7 and press **[x]**. Then graph Y_1 , Y_2 , and Y_3 . Are there similarities between Y_1 , Y_2 , and Y_3 ? If so, explain. _____

18. Based on the relationship of a person that fits a truly golden proportion, which equation would you use to find their height? _____

19. Find the height of a truly "golden" person whose navel height is 100 cm. _____

